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Comparison of shell and folding model Λ wavefunctions in p-shell hypernuclei

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Abstract : Similar Λ wavefunctions are found from different calculations of ground-state Λ binding energies in hypernuclei. At the same time, there is urgent need for Λ excited states and other data in order to better understand the Λ-N interaction.

Keywords : Hypernuclei, shell and folding model, Λ wavefunctions

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1. Introduction

The Λ-particle in nuclei provides an interesting additional probe for the understanding of the nuclear force, in view of SU (3) symmetry [1–4]. The cross section for the formation of Λ-hypernuclei decreases rapidly for higher A and the most interesting cases of Λ-hypernuclei occur in the p-shell region. Earlier, Kakkar *et al* [5] had noticed that calculated Hartree-Fock and harmonic oscillator wave functions gave very similar results in the restricted configuration space in the case of the p-shell for the nuclei ^{18}O, ^{18}F and ^{10}Be. At Aligarh, apart from semi-empirical studies of B_Λ in the ground and the excited states, and D_Λ in infinite nuclear matter [6,7] and of low energy Λ–p scattering [8], two different approaches, namely the shell-model approach [9–12] and the folding-model approach [13,14], have been employed for the study of hypernuclear bound states.

The most noteworthy feature of the shell model approach was that a purely central, effective, spin and state (*i.e.* parity)-dependent Λ–N potential, without any spin-orbit or tensor forces, could successfully reproduce the B_Λ data in the ground state, if core size was allowed to vary within experimental limits for different mass nuclei [9]. If, in addition, a three-body ΛNN force is also taken into account, the state-spin-dependence of the effective Λ–N potential is significantly reduced. This is consistent with the expected features of the Λ–N interaction, as evidenced from strangeness conservation considerations, and the various quark-exchange studies of the hypernuclear systems. The centre-of-mass (CM) corrections were suitably incorporated in the calculation, as were exchange terms in calculating the matrix elements. However, core polarisation effects were estimated to be small, and were found to be masked by uncertainities in the core radii.

In the folding model approach, ground state B_Λ data is reproduced for s- and p-shell Λ-hypernuclei, assuming either a zero-range or a 2π-exchange-range Λ–N force where $\rho^{2/3}(r)$ dependence is invariably assumed, as is done generally in low energy nuclear physics, and in optical model studies of scattering of light nuclear projectiles. The exact form of the effective Λ–N potential assumed is $V(r) = -V_0\rho(r)\left(1-\beta\rho^{2/3}(r)\right)$, with V_0 and β as parameters, (in addition to the range of Λ–N interaction, if the same is not taken as equal to the nucleon size as a first approximation). The eigenvalue problem is then solved exactly with the help of a computer.

We have plotted and compared the wave functions as obtained from the above two approaches separately, for three p-shell Λ-hypernuclei, namely those with $^7_3$Li, $^{10}_5$B and $^{14}_7$N cores respectively, and have found that for good-fit B_Λ values on the above two models [9,13], the respective wave functions show a high degree of overlap. Thus, our study makes us understand why calculations of B_Λ done under different sets of reasonable premises and assumptions lead to similar results.

2. Calculations, results and discussion

We consider only those hypernuclei that were studied by both Mujib *et al* [9] and Ahmad *et al* [13].

In the former study, the oscillator size parameter b was fixed from the rms radius of the relevant core nuclei as obtained from electron scattering experiments. Then the size parameter b_Λ is determined separately for each hypernucleus by minimizing the Λ-binding energy, as described in detail in [9]. The value of b is taken to be the same for s-shell as well as for p-shell nucleons, so that $\rho(r)$ is given by

$$\rho(r) = (1/Z)\left[Z_s\left(4/\pi^{1/2}b^3\right)\exp\left(-r^2/b^2\right)\right.$$
$$\left. + Z_p\left(r^2/b^2\right)\left(8/\pi^{1/3}b^3\right)\exp\left(-r^2/b^2\right)\right], \tag{2.1}$$

where Z_s and Z_p are the numbers of protons in the s- or the p-shell respectively. The Λ–N potential taken, is spin- as well as state-dependent, and is taken as

$$V_{\Lambda N} = \left[(1/4)\left(U_s^l + 3U_t^l\right) - (1/4)\left(U_s^l - U_t^l\right)\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N\right] f(r) \tag{2.2}$$

with $$f(r) = \exp\left(-r^2/\alpha_\Lambda^2\right)\pi^{3/2}\alpha_\Lambda^3 \text{ for the Gaussian,} \tag{2.2a}$$

and $$f(r) = \delta(\vec{r}) - (I_2/3!I_1)h^{-2}\left[p^2\delta(\vec{r}) + \delta(\vec{r})p^2 - 2\vec{p}\cdot\delta(\vec{r})\cdot\vec{p} + \cdots\right] \tag{2.2b}$$

for the Skyrme case,

where $$I_n = \int_0^\infty r^{2n} f(r)dr, \quad n = 1, 2, 3, \ldots .$$

The Λ is assumed to be in the s-state so that

$$r^2|\psi_\Lambda|^2 = \left(4r^2/\pi^{1/2}b_\Lambda^3\right)\exp\left(-r^2/b_\Lambda^2\right). \tag{2.3}$$

In the latter study, V_Λ is assumed to be density-dependent but otherwise spin and state independent. $\rho(r)$ is taken to correspond to the oscillator form [15]

$$\rho(r) = \rho_0\left[1 + \alpha(r/a)^2\right]\exp\left(-r^2/a^2\right),$$

$$\alpha = \alpha_0 a_0^2 \Big/ \left[a^2 + (3/2)\alpha_0\left(a^2 - a_0^2\right)\right], \qquad a_0^2 = \left(a^2 - a_p^2\right)A\Big/(A-1),$$

$$\alpha_0 = (Z-2)/3, \qquad a_p^2 = (2/3)\left\langle r^2\right\rangle_{\text{proton}},$$

and $$\left\langle r^2\right\rangle_{\text{proton}} = 0.856\,\text{fm}^2. \tag{2.4}$$

As a and α are interconnected, and ρ_0 can be obtained from the formula

$$\rho_0 = A\Big/\left[\pi^{3/2}a_0^3(1 + 1.5\alpha)\right], \tag{2.5}$$

it is clear that we can obtain either α or a by calculating the rms radius from the usual formula

$$\left\langle r^2\right\rangle = \int_0^\infty r^4\rho(r)\,dr \Big/ \int_0^\infty r^2\rho(r)dr \tag{2.6}$$

and comparing it with the rms radius value as given experimentally.

The form of density-dependence of V_Λ is taken as

$$V_\Lambda(r) = -V_0\rho(r)\left[1 - \beta\rho^{2/3}(r)\right], \tag{2.7}$$

where $\rho(r)$ is assumed to be the same as the charge density distribution. B_Λ for 8 hypernuclei are χ^2-reproduced [13] to give the values of parameters as V_0 = 382.87 MeV

fm^3, $\beta = 1.85$ fm^2. We can take $\rho(r)$ to be charge densities of the harmonic oscillator form and fix the parameters α by fitting the calculated rms radii of core nuclei to the experimental rms radii as taken by Mujib *et al* [9]. ρ_0 is fixed by the value of A. V_0 is fixed by fitting the value of B_Λ obtained by numerical integration of the Schrödinger equation to the experimental value of B_Λ for the particular nucleus. The wavefunction $r\psi_\Lambda$, so obtained, for each nucleus, is normalized.

The overlap integral can be calculated for the values of wave functions from the two approaches.

The results obtained are shown below :

The Gaussian and Skyrme potential parameters and the experimental B_Λ and rms radius values, as also the values of bound b_Λ relevant to the study, are taken as

$$U_s^0 = -543.66 \text{ MeV fm}^3, \qquad U_t^0 = -225.56 \text{ MeV fm}^3,$$

$$U_s^1 = -1950.23 \text{ MeV fm}^3 \quad \text{and} \quad U_t^1 = 946.89 \text{ MeV fm}^3,$$

$$\alpha_\Lambda = 1.26 \text{ fm for the Gaussian,}$$

and

$$U_s^0 = -480.60 \text{ MeV fm}^3, \qquad U_t^0 = -299.45 \text{ MeV fm}^3,$$

$$U_s^1 = -1142.22 \text{ MeV fm}^3 \quad \text{and} \quad U_t^1 = 1044.29 \text{ MeV fm}^3,$$

$$a = \left(I_{2/3!}I_1\right) = 0.33 \text{ for the Skyrme case,}$$

and as per Table 1.

Table 1. The experimental B_Λ and rms radius values, alongwith the values of oscillator size parameters b and b_Λ.

Sl. No.	Hypernuclei	$B_\Lambda^{\text{expt.}}$ (MeV)	(rms radius)$^{\text{expt}}$ (fm)	b (fm)	b_Λ (fm)
1	${}_3Li_\Lambda^8$	5.58 ± 0.03	2.29 ± 0.03	1.705	1.715
2	${}_5B_\Lambda^{11}$	10.24 ± 0.05	2.45 ± 0.12	1.743	1.774
3	${}_7N_\Lambda^{15}$	13.56 ± 0.15	2.45 ± 0.05	1.65	1.929

For the folding model, β was taken as 1.85 fm^2 [13], whereas the values of B_Λ and V_0 obtained were as given in Table 2 where the last column shows the percentage overlap between wavefunctions calculated from the shell model and the folding model approaches respectively, as described in detail in the text.

Table 2. The calculated B_Λ and V_0 on the folding model, and the percentage overlap of the two Λ wavefunctions.

Sl. No.	Hypernuclei	B_Λ^{cal} (MeV) (Folding model)	V_0 (MeV fm^3)	% overlap of the two Λ wavefunctions
1	${}_3Li_\Lambda^8$	6 8	385.91	90.3
2	${}_5B_\Lambda^{11}$	10.24	384 18	96.0
3	${}_7N_\Lambda^{15}$	13.6	390.92	95 2

Plots of $r\psi_\Lambda$ and $\rho(r)$ for the two approaches considered are shown in Figures (1–3) for the three hypernuclei : ${}_3Li_\Lambda^8$, ${}_5B_\Lambda^{11}$ and ${}_7N_\Lambda^{15}$.

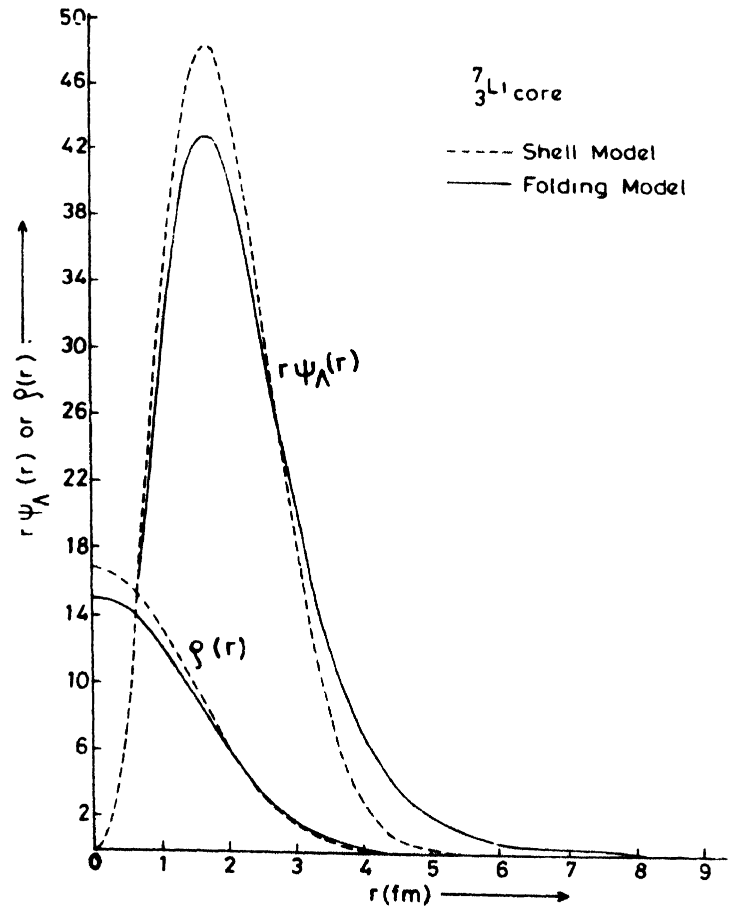

Figure 1. Λ wave functions and nuclear densities from shell model (broken line) and folding model (solid line) calculations. ${}_3^8Li_\Lambda$.

The nuclear core densities obtained in the two cases differ from each other near $r = 0$ but this is the region where the wavefunction $r\psi_\Lambda$ tends to zero in both the cases. On the other hand, where the wavefunctions, $r\psi_\Lambda$, have their maxima, the two nuclear core densities almost coincide.

As the snell model approach involves spin as well as state dependence of Λ–N potential, in addition to CM correction, there are contributions to kinetic and potential energies as spin-independent or spin-dependent parts from relative $l = 0$ and relative $l = 1$ states, as reported by Mujib *et al* [9]. In the folding-model approach, the kinetic energy decreases with A as expected (the simple square well implies that kinetic energy must vary as $A^{-5/3}$).

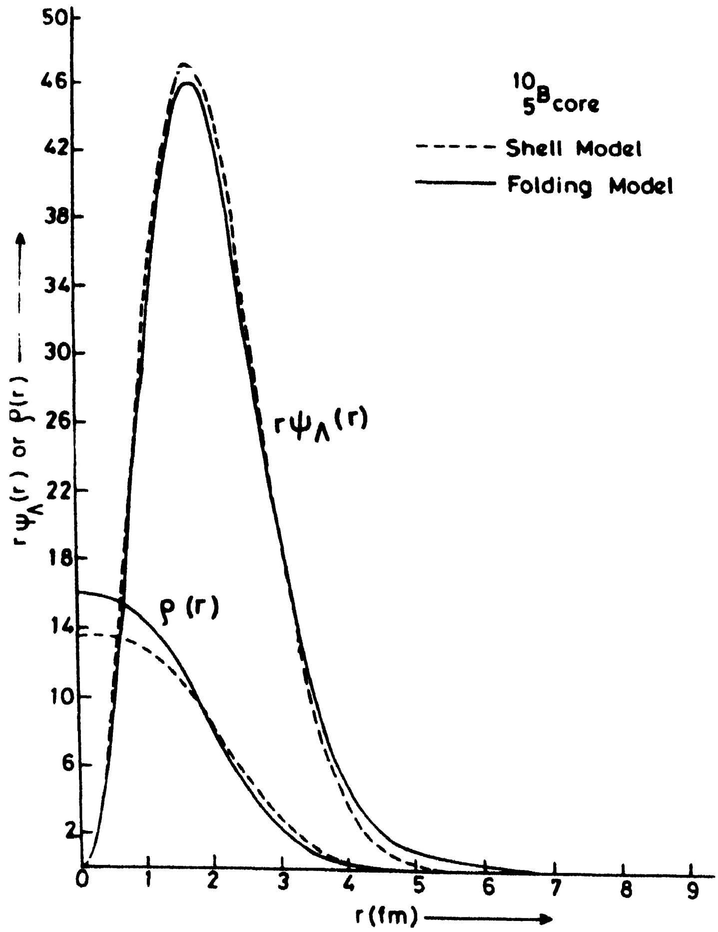

Figure 2. Λ wave functions and nuclear densities from shell model (broken line) and folding model (solid line) calculations. ${}^{11}_{5}B_\Lambda$.

It is reassuring from this study to find very similar Λ wave functions for the different calculations. It imparts a measure of confidence in these different studies. However, at the same time, it also points to the fact that ground state B_Λ data do not provide good test of a theory. One will have to look at the excited states and other data.

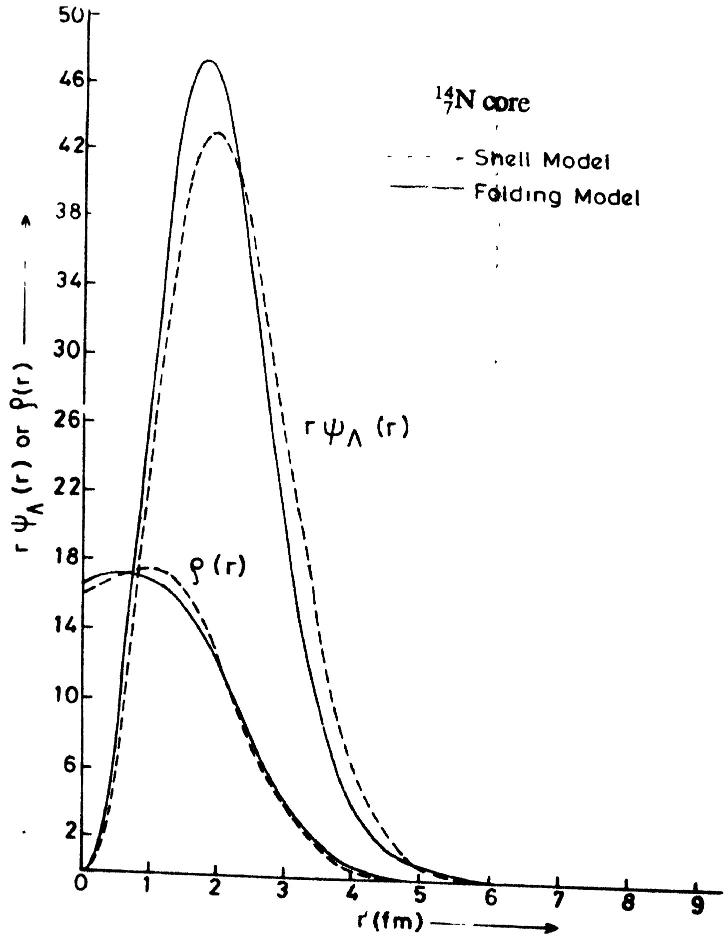

Figure 3. Λ wave functions and nuclear densities from shell model (broken line) and folding model (solid line) calculations. $^{15}_{7}N_\Lambda$.

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